Generative Models for NLP

Introduction and Word Embeddings

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Notes

Outline

Notes

NLP

Various tasks that *process data encoded in natural language* (NL)

speech recognition, text classification, NL understanding, and NL generation. . .

At the crossroad of :

- Linguistics
- Computer Science (formal languages, automata, graphs. . .)
- Logic
- Machine Learning (probability/optimization/statistics)
- Artificial Intelligence
- Cognitive Sciences

Notes

Language as a Symbolic System

Language

a system that allows a speaker (writer) to communicate (among other functions)

Elements of this system are discrete (symbols)

- speech/gestures/writings are transctiptions of this system
- if continuous, no writing system (discretization) possible without major loss of meaning

Problems for Machine Learning

- Requires many different symbols, some very frequent some very rare
- Discrete units make gradient descent impractical

Notes

Why it's difficult

Ambiguity at all levels

- *saw, duck, her, round, like*, (lexical amb. of words)
- *I saw her duck* (lexical amb. in context)
- *John eats a pizza with a fork* vs. *John eats a pizza with an egg* (syntactic amb., attachment)
- *A computer that understands you like your mother* (amb. syntaxique, rattachement)
- *I seek a unicorn* (semantic amb. existential quantification)
- *avocat pourri* → *dirty lawyer, rotten avocado* (expressions and translation)

Yes but... we understand each other!

- Context may help, but how?
- world knowledge, social convention, statistics may also help
- other modalities (vision, touch...)

Great variability

sometimes very ambiguous... in general simple (simple enough for humans to learn!)

Notes

What we will discuss this year

Session 1: Word Embeddings

<2026-01-09 Fri>

- What are the units? how do we represent them?

Session 2: Language Models

<2026-01-16 Fri>

- How do we represent words in context
- How can we represent sequences and generate texts

Session 3: RNN/Transformers and Attention

<2026-01-23 Fri>

- Attention mechanism

Session 4: Fine-tuning LLMs with RLHF and DPO

<2026-01-30 Fri>

- How to incorporate application preferences in LLMs

Session 5: Latent Variable Models for text: Diffusion Language Models

<2026-02-06 Fri>

Notes

Words / Vocabulary / Lexicon / Index

Definitions

- word (token) = atomic element : *dog*, *car*, *the*, *Trump* ...
  - 🤔 what about letters?
- Vocabulary V finite set of all accepted words
- add a *special token* `UNK`
  - representing all the things in texts that are not words (*ie qwerw3y*)
  - representing all the words that may be missing in V
- Lexicon L_V finite set of words or `UNK` : $L_V = V \cup \{\texttt{UNK}\}$
- Language $\mathcal{L}_V$ is the set of strings on the lexicon $L_V^* = \bigcup_{i=0}^{\infty} L_V^i = \{\varepsilon\} \cup L \cup LL \cup \dots$
- To each element of the lexicon L correponds a unique integer, its *index*, between 0 (for `UNK`) and $|\mathrm{V}|$.

Notes

Word sense – Word meaning

Fundamental issue in NLP: assign a meaning to words

Different point of views:

- Linguistics: link *signifier* (sound/writing) and the *signified* (the thing/the meaning)
  - not really mechanizable
- Ontology (data/knowledge base): organize a hierarchy of concepts and terms on a semantic basis
  - do not really say anything about usage (how to use words)
- CS/NLP: pragmatic approach, meaning depends on context defined by application
  - EX: if a vocal assistant must perform the same when user says *play the Beatles* or *put the Beatles on* or *read the Beatles*, in this context *play,read,put on* must have the same meaning/representation
  - Moreover their meaning is to *launch the player*

Notes

Multi-dimensional representation (word vectors)

Assume contexts are countable (and sparse)

Then, if meaning depends on context, and if context can change, we can represent words as vectors with one dimension by context. 🤯

Synonymity between two words

- Compute *distance* / *inner product* between their vectors.
- *Partial* synonyms, depend on contexts 🤔
- Geometrical Meaning!

Geometrical Meaning

- Represent words in dense vectors $\mathbb{R}^d$
- Distance/similarity interpreted as semantic relatedness (*eg* synonyms)

Eg. assuming that *play* and *launch* are closer semantically than *play* and *clean*, we have:

$$\|V(\text{play}) - V(\text{launch})\|_2^2 \leq \|V(\text{play}) - V(\text{clean})\|_2^2$$

Notes

(A long time ago. . .)

Very different representations in NLP/CL!

Lexicon elements used to be considered as unique by default. Equivalently, they were represented by *one-hot* vectors in $\{0,1\}^{|L_V|}$. For instance:

play	$[000000000010000000]$
read	$[001000000000000000]$
stop	$[000000000000000100]$

All tokens were orthogonal, (inner product zero, distance $\sqrt{2}$), whether they had the same meaning or not. Geometry meant nothing

Notes

Contextual (or Distributional) Representation

Contexts as neighbouring words

You shall know a word by the company it keeps. (J. R. Firth, 1957)

An old idea in linguistics, used in NLP since 90s/00s
The meaning of a word can be inferred from its *usage*, therefore from the sequence of token in which it appears.
A word is represented by a set of contexts

Example

For instance for *théorie* in a French text corpus:

... Ezion qui le mettaient en	théorie	à l' abri de soucis ...
... essais assez pointus sur une	théorie	mathématique quelques articles réunis trois ...
... poète météo dépendant pas de	théorie	.

théorie = { ... Ezion qui le mettaient en, à l' abri de soucis ..., ... essais assez pointus sur une, mathématique quelques articles réunis trois ..., ...}

Observation

- The more contexts 2 words have in common, the more they share meaning.
- This is not scalable ⇒ contexts must be compressed

Notes

Word Vectors x Distributional Representation = Word Space Models

Vector similarity is the only information present in Word Space: semantically related words are close, unrelated words are distant. (H. Schütze, 1993)

In the 90s several methods appear, based on word co-occurrences, with dimension reduction for efficiency and densification.

Big Picture

1. Associate each lexicon elt e to a vector count v_e of length $|L_V|$, initialized to zero
2. From a set of documents
   - count $n_{e,e'}$ for each lexelt e, the number each e' occurs in the neighbourhood of e
   - filter: discard tokens such as *a, the*..., limit to a window of k tokens around e,
   - set $v_e[id(e')] = n_{e,e'}$
3. Concatenate all vectors $v_e, \forall e$ into a co-occurrence matrix M
   - perform some normalization (*eg* TF/IDF or a variant)
4. M is big and sparse
   - use (Truncated) Singular Value Decomposition to get matrix M'
   - such as M'_e is a dense vector of dimension $d << |L_V|$

Notes

Word Vectors x Distributional Representation = Word Space Models (2)

Usage

To decide whether two words are similar semantically, DR usually uses *normalized* inner product also called *cosine similarity*:

$$sim(x,y) = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2}\sqrt{\sum_i y_i^2}} = \frac{x \cdot y}{(\sqrt{x^2})(\sqrt{y^2})} = \frac{\langle x, y \rangle}{|x| \times |y|}$$

(🤔 what are the min/max values of *sim*)

Issues with distributional approach

- normalisation, scaling → compression (matrix dimension reduction)
- not incremental → need to retrain from scratch for new data

Notes

Word2Vec (2)

Algorithm to parametrize word vectors

- implements Firth's principle, related to SVD methods (1)
- simple idea (but implementation may be *tricky*)

Not a *Neural Network* but

- the resulting vectors will be used as input by NNs later
- learning performed via gradient descent
- probabilistic modelling (and approximation)

A little old...

- More powerful recent methods
- ... but `Word2Vec` is simple and implements Firh's idea quite directly

Notes

WordVec Skip-gram Model

Principle

1. Assume we have a corpus of texts (big, >1M words) for training;
2. We set vocabulary V, out of vocabulary (OOV) words are replaced by token `UNK`;
3. At each position t of a text, we write token c at position t and its neighbouring tokens O_c (words in a window of size m around t). We assume a factorized probability to generate neighbouring tokens: $p(O_c|c;\theta) = \prod_{o \in O_c} p(o|c;\theta)$
4. From the computation of similarity between center word vector v_c and context word vectors v_o, we will define probability $p_\theta(o|c)$;
5. Training objective: maximize parameters for likelihood: $\prod_c p(O_c|c;\theta)$.

Loss

Maximizing the log-likelihood amounts to the following loss:

$$\max_\theta \log \prod_t \prod_{o \in O_t} p(o|c_t;\theta) = \max_\theta \sum_t \sum_{o \in O_t} \log p(o|c_t;\theta)$$
$$= \min_\theta \sum_t \sum_{o \in O_t} - \log p(o|c_t;\theta)$$

Notes

Word2vec: Neighbours and Probability

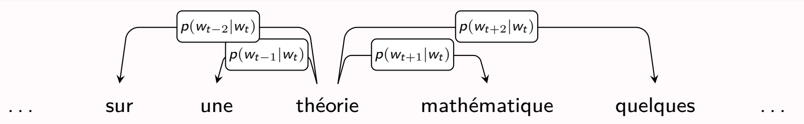
Window

- Given a position t in text, we write w_t, the token at this position
- A window of size m centered at position t, noted O_t consists of all words $w_i, \forall (t-m) \leq i \leq (t+m)$

Word2vec defines probability distributions

- Probability for all words w' to be in a window (of predefined size m) of a specific word w
- In the remainder, we call $p(w'|w)$ *neighbourhood probability*

Example



Notes

Word2vec : Maximum Likelihood Estimation (MLE)

General method to cast a ML problem (*eg* optimized by SGD) into a probabilistic framework. Probabilities are computed from scores given by a network with parameters θ

Definition (Likelihood)

A function returning a corpus probability (assume iid observations) from given parameters:

$$V(\theta) = \prod_{t \in \mathcal{T}} p(O_t | w_t; \theta) = \prod_{t \in \mathcal{T}} \prod_{t-m \leq j \leq t+m} p(w_j | w_t; \theta)$$

Intuition

Observations must be more probable that any event that we did not observed, and because observations *happened* they must have a probability 1. We want to *maximize* V:

$$\theta^* = \arg\max_\theta V(\theta) = \arg\max_\theta \prod_t \prod_{t-m \leq j \leq t+m} p(w_j | w_t; \theta)$$

Notes

Word2vec: From MLE to continuous optimization

Product → Sum

$V(\theta)$ is a product: difficult to optimize. $\log$ is monotone so we can write:

$$\theta^* = \arg\max_\theta \log V(\theta) = \arg\max_\theta \sum_t \sum_{t-m \leq j \leq t+m} \log p(w_j | w_t; \theta)$$

Minimization

We prefer minimizing a loss function:

$$\theta^* = \arg\min_\theta - \log V(\theta) = \arg\min_\theta \sum_t \sum_{t-m \leq j \leq t+m} - \log p(w_j | w_t; \theta)$$

Finally $-\log V(\theta)$ is a loss function

- positive
- zero when no error

Notes

Parametrization of probabilities

Similarity between words

- We want to measure similarity between token w_t et context token w_o
- Similarity of their associated vectors $\rightarrow$ inner product
- Issue with inner product: $v_1^\top v_2$ is maximal when $v_1 = v_2$. we want to avoid trivial solution: $v_i = v_j \forall i, j$
- Solution: define 2 vectors per token mot m:
  1. v_m when m is the center position of the window
  2. u_m when m is a context position in the window.

From similarity to proability

- in $p(w'|w)$, use inner product $u_{w'}^\top v_w$
- we use softmax:

  for a vector $a = [a_1 \dots a_n]$ softmax return a vector:

$$\text{softmax}(a)[i] = \frac{\exp(a[i])}{\sum_j \exp(a[j])}$$

Notes

Parametrization of probabilities (2)

Inner product + softmax =

$$p(w'|w) = \frac{\exp(u_{w'}^\top v_w)}{\sum_{w''} \exp(u_{w''}^\top v_w)}$$

Softmax implements the idea that the more context w' is similar to center w, that the higher $u_{w'}^\top v_w$ the more probable it is to appear in a window.
How to implement?

Notes

First implementation

In lab, you will code an efficient batched version

```
import torch

class Word2Vec(torch.nn.Module):
    def __init__(self, corpus, lexicon_size, vec_size):

        # 2 tables to define from data:
        #word2idx table token -> index
        #idx2word table index -> token

        U = torch.nn.Embedding(lexicon_size, vec_size)
        V = torch.nn.Embedding(lexicon_size, vec_size)

    # returns the sim. score for all token as contexts of w_c
    def forward(self, idx_c):
        # inner product with all vectors in U
        logits = self.U.weight @ self.V(idx_c)
        return logits
```


Notes

First implementation (2)

```
import torch

def train(net, opt, train_set, nb_epoch):

    loss_fn = torch.nn.CrossEntropyLoss()
    # to adapt for lab sessions
    # to deal with batched input
    for epoch in range(nb_epoch):
        train_set.shuffle()
        for (o,c) in train_set:
            logits = net(c)
            loss = loss_fn(logits, o)
            loss.backward()
            opt.step()
            opt.zero_grad()

w2v = Word2Vec(corpus, 10000, 20)
# TODO: conversion between data and training set
#optimizer = ...
train(w2v, optimizer, train_set, 20)
```


Notes

Some issues with this version

Softmax on vocabulary

$$p(w'|w) = \frac{\exp(u_{w'}^\top v_w)}{\sum_{w''} \exp(u_{w''}^\top v_w)}$$

Will not scale for large vocabularies:

- complexity (time/memory) (denominator)
- rounding errors

In the *real* Word2vec

- many modifications to improve efficiency
- less rounding errors

Coming up next:

- a better implementation
- interpretation/explanation of the implementation
- beaucoup d'équations 🤯

Notes

MLE and Cross Entropy

In `Pytorch` learning with MLE uses a function called *cross entropy* 🤔

Definition (Entropy of a distribution)

a measure of the *randomness* of a distribution

$$H(p) = -\sum_{c \in \mathcal{C}} p(c) \log p(c)$$

- $H(c) \geq 0$
- If p uniform (*ie* random) $H(p) = \log(|\mathcal{C}|)$
- If p deterministic ($p(c_i) = 1$), $H(p) = 0$

Notes

MLE and Cross Entropy (2)

Definition (Conditional Entropy / Cross Entropy)

Allows to compute a kind of *distance* between 2 distribution 🤔

$$\mathtt{CE}(q, p) = -\sum_c q(c) \log p(c)$$

Definition (Empirical distribution p_e)

- *distribution* of observed data $s = c_i$ always 0/1
- for a multinomial, if for $p_e(c_i) = 1$ then $\forall j \neq i, p(c_j) = 0$
- (generalizezs to mean for several examples)

Equivalence

Let us compute the cross-entropy between:

- q the empirical distribution
- p the distribution computed by our model

Notes

MLE and Cross Entropy (3)

$$\begin{aligned}
\mathtt{H}(q, p) &= -\sum_c q(c) \log p(c) \\
&= -q(c_i) \log p(c_i) - \sum_{c \neq c_i} q(c) \log p(c) \\
&= -1 \times \log p(c_i) - \sum_{c \neq c_i} 0 \times \log p(c) \\
\mathtt{H}(q, p) &= -\log p(c_i)
\end{aligned}$$

We recover $-\log p(x)$, the loss we defined for MLE 🤯

Notes

Cross-Entropy in Pytorch (1)

```
# classifier for input of size 10, 5 classes
# 8 hidden neurons (cf. DATA MINING)
net = MLP(10,[8],5)

# let us pretend the following tensor contains
# the input for 3 examples
inputs = torch.rand(3,10)

# we obtain the 5 scores for all 3 examples
preds = net(inputs)

# let us suppose that the correct classes were:
empirical = torch.tensor([0,2,1], dtype=torch.long)

loss_function = torch.nn.CrossEntropyLoss()

# note that we do not compute log softmax (inside CE)
loss = loss_function(preds, empirical)

loss.backward() # and the rest...
```


Notes

Cross-Entropy in Pytorch (2)

After training, prediction:

```
# let us pretend the following tensor contains
# the input for 3 examples
inputs = ...

# we obtain the 5 scores for all 3 examples
predictionss = net(inputs)

#apply the argmax for each line:
outputs = torch.argmax(predictionss, dim=1)
```

pas de *cross-entropy*, pas de *softmax* : pourquoi ?

Notes

Constrastive Sampling Approach of Mikolov et al. (1)

Softmax inefficient → alternative method to train word vectors.

From *word* probability to *class* probability

Define probability $p(D = d|w', w)$ for 2 classes d:

- $d = 1$ if w' is a word that belongs to a possible context for w
- $d = 0$ otherwise
- of course we have: $p(D = 0|w', w) = 1 - p(D = 1|w', w)$

How to model binomial distribution? (2 classes)

- softmax possible but...
- sigmoid more *natural* $\sigma(x) = \frac{1}{1+\exp(-x)}$
- combine sigmoid and similarity:

$$p(D = 1|w', w) = \frac{1}{1 + \exp(-u_{w'} \cdot v_w)}$$

Notes

Constrastive Sampling Approach of Mikolov et al. (2)

What we want to do

- train vector for tokens from a corpus organised in sentences...
- ... by using a loss function that involves a *similarity* beetween word vectors
- efficient: we want to avoid iterating through the lexicon

Maximum Likelihood Estimation

For each position t in the training corpus:

- we want to correctly classify (w', w_t) (to 0 or 1) for all w'
- we write $k(c, w)$ the class of context candidate c for token w (set to 0 or 1) in the training set. MLE amounts to:

$$\theta^* = \max_\theta \sum_t \mathbb{E}_{w \sim P_{w_t}(.)}[\log p(D = k(w, w_t)|w, w_t)]$$

Notes

Constrastive Sampling Approach of Mikolov et al. (3)

Maximum Likelihood Estimation: Decompose on positive/negative class expectations:

$$\theta^* = \max_\theta \sum_t \mathbb{E}_{w \sim P_{w_t}(.)}[\log p(D = k(w, w_t)|w, w_t)] \tag{1}$$

$$= \max_\theta \sum_t \sum_w P_{w_t}(w) \log p(D = k(w, w_t)|w, w_t) \tag{2}$$

$$= \max_\theta \sum_t \sum_w \sum_{i=0}^{1} P(D = i) \times P_{w_t}(w|D = i) \log p(D = k(w, w_t)|w, w_t) \tag{3}$$

$$= \max_\theta \sum_t \sum_{i=0}^{1} \sum_w P(D = i) \times P_{w_t}(w|D = i) \log p(D = k(w, w_t)|w, w_t) \tag{4}$$

$$= \max_\theta \sum_t \sum_{i=0}^{1} P(D = i) \sum_w P_{w_t}(w|D = i) \log p(D = k(w, w_t)|w, w_t) \tag{5}$$

$$= \max_\theta \sum_t \sum_{i=0}^{1} P(D = i) \mathbb{E}_{w \sim P^i_{w_t}(\cdot)}[\log p(D = k(w, w_t)|w, w_t)] \tag{6}$$

$$= \max_\theta \sum_t \sum_{i=0}^{1} P(D = i) \mathbb{E}_{w \sim P^i_{w_t}(\cdot)}[\log p(D = i|w, w_t)] \quad \text{why ?} \tag{7}$$

🤯

Notes

Constrastive Sampling Approach of Mikolov et al. (4)

Add the following assumptions:

There are more negative examples than positive examples

- $\exists \kappa = 1, 2, \ldots, N$ such that $P^-(D = 0) = \kappa \times P^+(D = 1)$
- in other words, the negative class is κ times more likely than the positive class

Expectations can be approximated by sampling efficiently (Monte-Carlo!)

For a position t, we want to maximize:

$$\mathbb{E}_{w \sim P^1_t(.)}[\log p(D = 1|w, w_t)] + \kappa \mathbb{E}_{w \sim P^0_t(.)}[\log p(D = 0|w, w_t)]$$

Expectations → Sampling:

$$\log p(D = 1|w^+, w_t)] + \sum_{i=1}^{\kappa} \log p(D = 0|w^{-i}, w_t)]$$

- w^+ drawn randomly from distribution $P^+_{w_c}$
- w^{-i} drawn randomly from ditribution $P^-_{w_c}$

Notes

Constrastive Sampling Approach of Mikolov et al. (5)

We also need to define distributions P^+ et P^-

For the positive class

draw randomly a token from the window around position t

$$P_t^+(w) = \begin{cases} \frac{1}{|O_t|} & \text{if } w \in O_t, \\ 0 & \text{otherwise} \end{cases}$$

For the negative class

Use the frequency of a word in training set as probability:

$$P_t^-(w) = \frac{\#w}{\sum_{w'} \#w'} = f(w)$$

In Mikolov's implementation, use a *flattened* version:

$$P_t^-(w) = \frac{\#w^{\frac{3}{4}}}{\sum_{w'} \#w'^{\frac{3}{4}}}$$

Notes

Constrastive Sampling Approach of Mikolov et al. (6)

Finally we obtain the Mikolov's loss:

$$\max \sum_t \left(\log \sigma(u_{w^+} \cdot v_{w_t}) + \sum_{i=1}^{\kappa} \log \sigma(-u_{w^-_i} \cdot v_{w_t}) \right)$$

- This is a maximization: we get the loss function by multiplying by -1

Notes

Constrastive Sampling Approach of Mikolov et al. (7)

A small trick to get a better model:

Subsampling

Do not take into account all the words in the training set:

- remove all words that appear less than n times (set $n = 2, 3, 4, 5 \ldots$)
- remove at random very frequent words in contexts. For each token w in the lexicon, eliminate w from O_t with probability:

$$P_d(w) = ReLU(1 - \sqrt{\frac{t}{f(w)}})$$

with $t = 10^{-5}$

- in words, words with frequency equal to or above t are always discarded.
- beware if we remove a word from context O_t, we still have to keep the size of the window $(2m)$: need to extend window boundaries

Notes

From MLE to Contrastive

Another way to look at contrastive estimation of word vectors

- a series of approximations/assumptions from the MLE model
- sheds new light on the relation between the two

Notes

From Definition to Implementation: Gradient

Let us derive the gradient for one example:

$$
\begin{aligned}
\nabla L(\theta; w', w) &= \nabla - \log p(w'|w;\theta) = \nabla - \log \frac{\exp(u_{w'}^\top v_w)}{\sum_{w''} \exp(u_{w''}^\top v_w)} \\
&= \nabla\Big(\log\big(\sum_{w''} \exp(u_{w''}^\top v_w)\big) - u_{w'}^\top v_w \Big) \\
&= \nabla\Big(\log\big(\sum_{w''} \exp(u_{w''}^\top v_w)\big)\Big) - \nabla(u_{w'}^\top v_w) \\
&= \frac{\nabla \sum_{w''} \exp(u_{w''}^\top v_w)}{\sum_{w''} \exp(u_{w''}^\top v_w)} - \nabla u_{w'}^\top v_w \\
&= \frac{\sum_{w''} \nabla \exp(u_{w''}^\top v_w)}{\sum_{w''} \exp(u_{w''}^\top v_w)} - \nabla u_{w'}^\top v_w \\
&= \frac{\sum_{w''} \exp(u_{w''}^\top v_w) \nabla u_{w''}^\top v_w}{\sum_{w''} \exp(u_{w''}^\top v_w)} - \nabla u_{w'}^\top v_w \\
&= \sum_{w''} p(w''|w;\theta) \nabla u_{w''}^\top v_w - \nabla u_{w'}^\top v_w \\
&= \mathbb{E}_{w'' \sim p(.|w;\theta)}[\nabla u_{w''}^\top v_w] - \nabla u_{w'}^\top v_w
\end{aligned}
$$

Notes

From Definition to Implementation: A New Objective

Recap: we want $\nabla L(\theta; w', w) = 0$

- equivalently: $\mathbb{E}_{w'' \sim p(.|w;\theta)}[\nabla u_{w''}^\top v_w] - \nabla u_{w'}^\top v_w = 0$
- But this gradient is also the gradient of:

$$F(\theta; w', w) = \mathbb{E}_{w'' \sim \widehat{p(.|w;\theta)}}[u_{w''}^\top v_w] - u_{w'}^\top v_w$$

- where $\widehat{x}$ means x is considered a constant (null gradient)

Conclusion We can learn with F instead of L (they have the same solution)

Wait... we still have to iterate over all words (expectation)

- Idea: sample only a few words, not all the lexicon

$$\text{With } w_k \sim p(.|w;\theta): \qquad F(\theta; w', w) \approx \frac{1}{K} \sum_{k=1}^{K} [u_{w_k}^\top v_w] - u_{w'}^\top v_w$$

with w_k sampled from p

Notes

From Definition to Implementation: Efficient Sampling

$$\text{With } w_k \sim p(.|w;\theta): \qquad F(\theta; w', w) \approx \frac{1}{K}\sum_{k=1}^{K}[u_{w_k}^\top v_w] - u_{w'}^\top v_w$$

Problems when sampling with p

To sample with p, we need to compute the denominator and iterate over the lexicon. . .

Solution

- Find an efficient approximation to $p(.|.;\theta)$
- `word2vec` uses the frequency of words in training corpus

$$P_u(w) = \frac{\#w}{\sum_{w'} \#w'}$$

- or a flattened version

$$P'_u(w) = \frac{\#w^{0.75}}{\sum_{w'} \#w'^{0.75}}$$

Notes

From Definition to Implementation: Rectification (1)

$$\text{With } w_k \sim P'_u(.): \qquad F(\theta; w', w) \approx \frac{1}{K}\sum_{k=1}^{K}[u_{w_k}^\top v_w] - u_{w'}^\top v_w$$

Recall : inner product $\approx$ vector similarity

- $u_{w'}^\top v_w$ must be positive (it's similar!)
- $u_{w_k}^\top v_w$ must be negative (it's not an observation!)

Use a Rectifier

We ignore the result of an inner product if it does not have the expected sign

- We could use `ReLU(x)=max(x,0)` (maybe. . .)
- in `word2vec` use a differentiable variant `softplus`

$$\text{sp}(x) = \log(1 + \exp(x))$$

- pay attention to minus signs - in the definition of F

Notes

From Definition to Implementation: Rectification (1)

$$F(\theta; w', w) \approx \frac{1}{K}\sum_{k=1}^{K}[-sp(-u_{w_k}^\top v_w)] - sp(u_{w'}^\top v_w)$$
$$\frac{1}{K}\sum_{k=1}^{K}[-\log(1+\exp(-u_{w_k}^\top v_w))] - \log(1+\exp(u_{w'}^\top v_w))$$
$$\frac{1}{K}\sum_{k=1}^{K}[\log(\frac{1}{1+\exp(-u_{w_k}^\top v_w)})] + \log(\frac{1}{1+\exp(u_{w'}^\top v_w)})$$
$$\frac{1}{K}\sum_{k=1}^{K}[\log(\sigma(u_{w_k}^\top v_w))] + \log(\sigma(-u_{w'}^\top v_w))$$

Can we recover contrastive estimation??

With $w_k \sim P'_u(.)$: $F(\theta; w', w) \approx \frac{1}{K}\sum_{k=1}^{K}[\log\sigma(u_{w_k}^\top v_w)] + \log\sigma(-u_{w'}^\top v_w)$

One last step to recover the signs of contrastive estimation

Notes

Conclusion

Word2vec

- Simple Idea, implements Firth's principle in an efficient way
- Probabilistic modelling : MLE/Constrative
- Many tricks to scale up:

Extensions

Dynamic Vectors: Word Vectors + Functions (NNs) to adapt Word vectors to each context (EIMO, BERT. . .)

Notes

Bibliography

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