Rewriting Modulo SMT Techniques for Parametric Analysis

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Work based on

Rewriting Logic Semantics and Symbolic Analysis for Parametric Timed Automata

Authors: Jaime Arias, Kyungmin Bae, Carlos Olarte, Peter Csaba Ölveczky, Laure Petrucci, Fredrik Rømming Authors Info & Claims

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Symbolic Analysis and Parameter Synthesis for Time Petri Nets Using Maude and SMT Solving

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Symbolic analysis and parameter synthesis for networks of parametric timed automata with global variables using Maude and SMT solving

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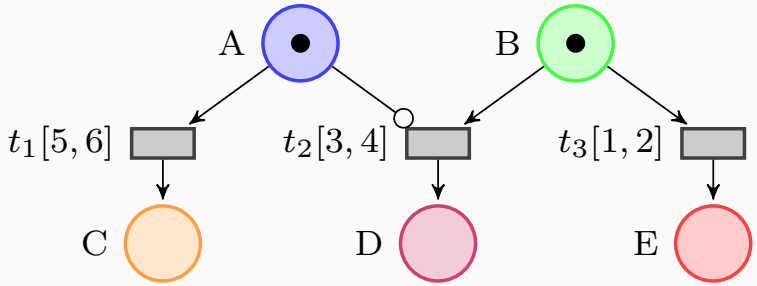
Joint work with: Jaime Arias, Kyungmin Bae, Peter Csaba Ölveczky, Laure Petrucci and Fredrik Rømming.

Motivation: Verification of real-time systems

- Timed automata
- Time Petri nets



Pros

- Decidable fragments
- Efficient verification procedures
- Extended with parameters.

Cons

1. No support for user-defined data types
2. No support for other forms of communication and dynamic object creation/deletion

Motivation: Verification of real-time systems

Rewriting logic / Maude

```
mod SYSTEM
  eq t = t' .
  rl l => r if C .
  ...
```

Pros

- Very expressive and general
- User-defined data types
- Large applications
- Executable specification
- Maude system: full LTL model checking, reachability, ...

Cons

- Most analysis problems are undecidable
- Explicit-state analysis of real-time theories is unsound for dense time

Contributions

- Interpreter: Executable rewrite semantics for parametric timed Automata (PTA) and parametric time Petri nets with inhibitor arcs (PITPN)
- Sound and complete symbolic analysis (Rewriting + SMT)
- Analysis methods: Reachability, parameter synthesis, model checking
- New analysis:
  - Full LTL model checking
  - Executions with strategies
  - Synthesis also of initial markings
- Novel folding procedure (termination)
- Long term goal: Symbolic analysis of real-time rewrite theories

Outline

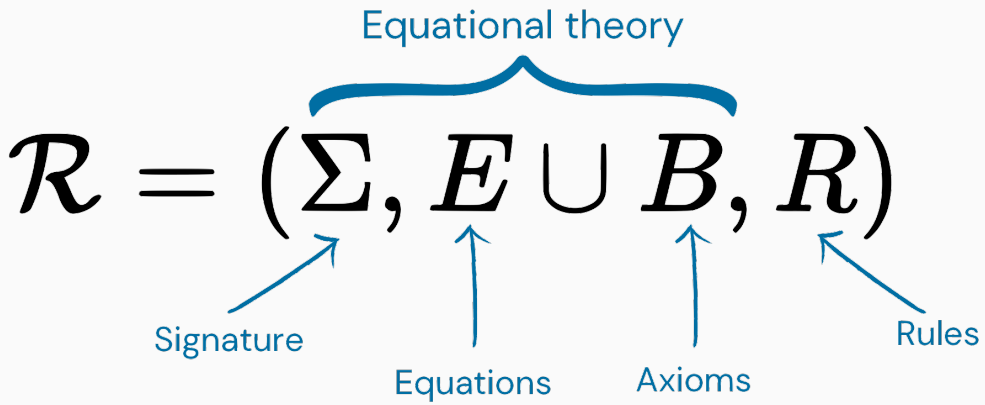
1. Rewriting logic
2. PTAs
3. PITPNs
4. Back to PTAs
5. Concluding Remarks

Plan

1. **Rewriting logic**
2. PTAs
3. PITPNs
4. Back to PTAs
5. Concluding Remarks

Rewriting Logic

- Computational logic: concurrent computation + logical deduction



- States are terms modulo $E \cup B$
- Rules (`crl l => r if C`) in R define system transitions

Rewriting Logic

- Equational theory (algebraic data types): defining states
- Rewriting rules: behavior of the system
- Executable specification

Maude

- A high-performance rewriting logic engine
- Executes admissible theories (confluence and termination of *E*, coherence of *R* w.r.t. *E*, ...)
- Several generic formal analysis tools (rewrite, search, LTL model checker, narrowing, SMT, etc).

Plan

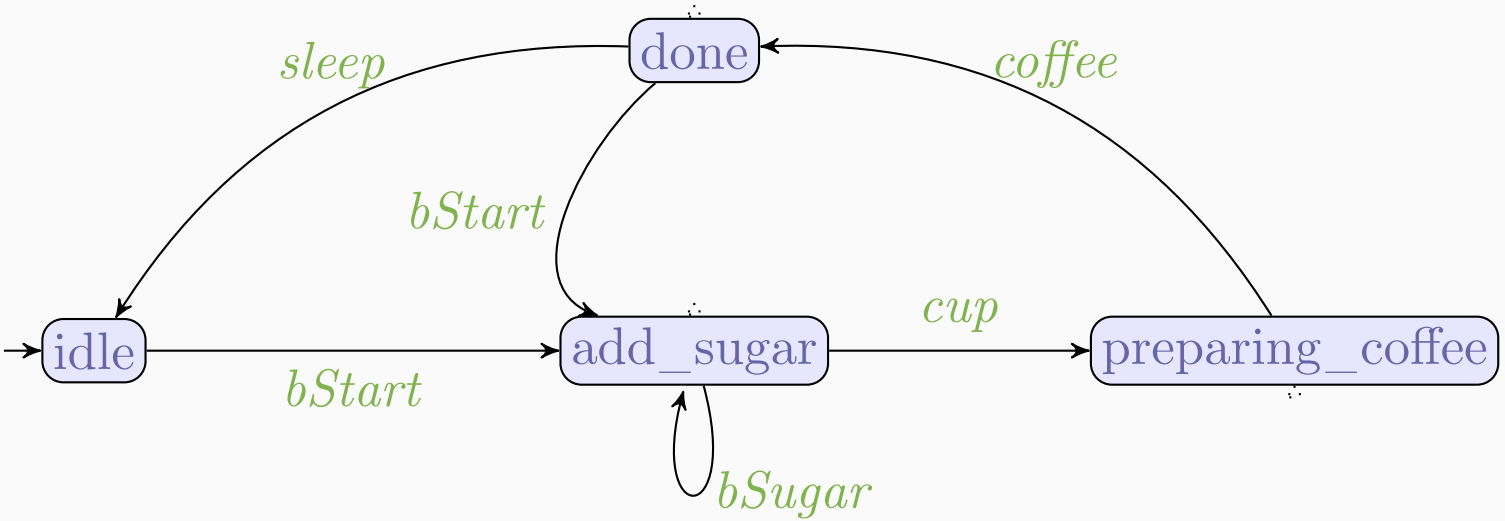
Parametric Verification

Models: not all the choices, components, response times, are known.

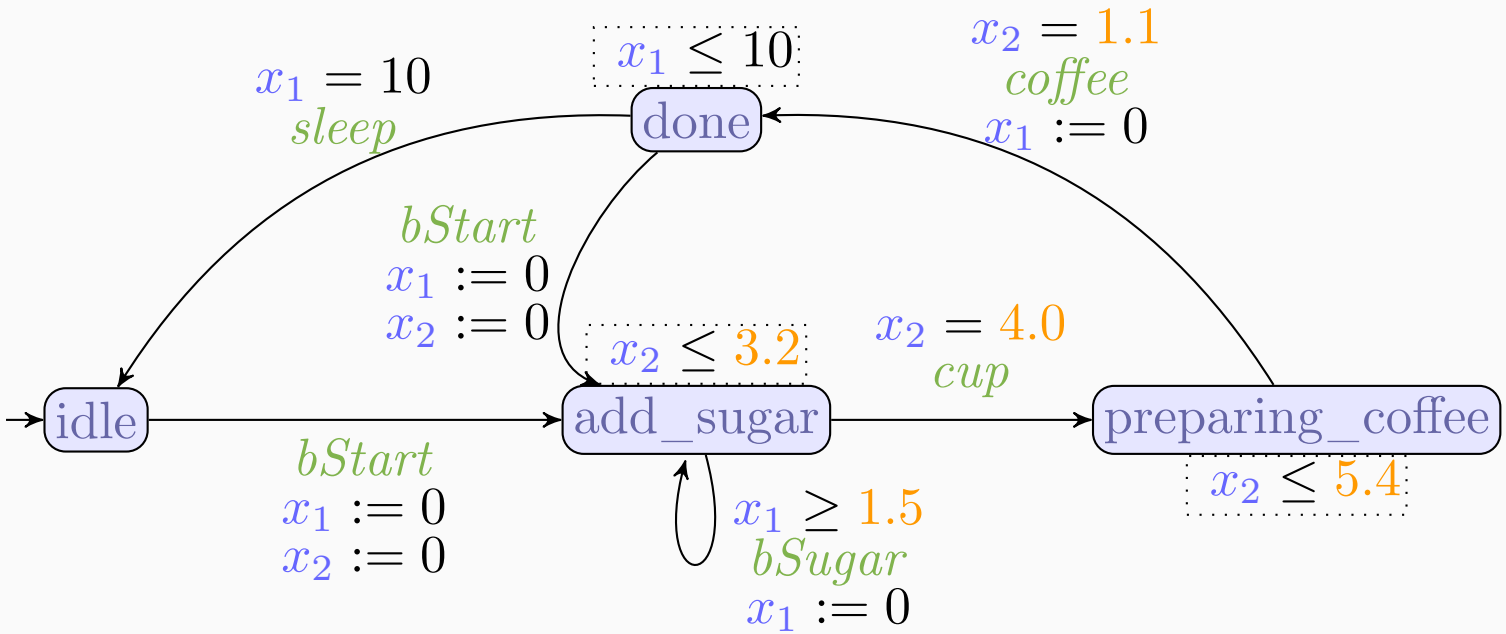
Parameters

- Flexibility
- Avoid verifying the system when the unknown components change
- Central problem: Parameter synthesis

Automata


done
sleep
coffee
bStart
idle
add_sugar
cup
preparing_coffee
bStart
bSugar

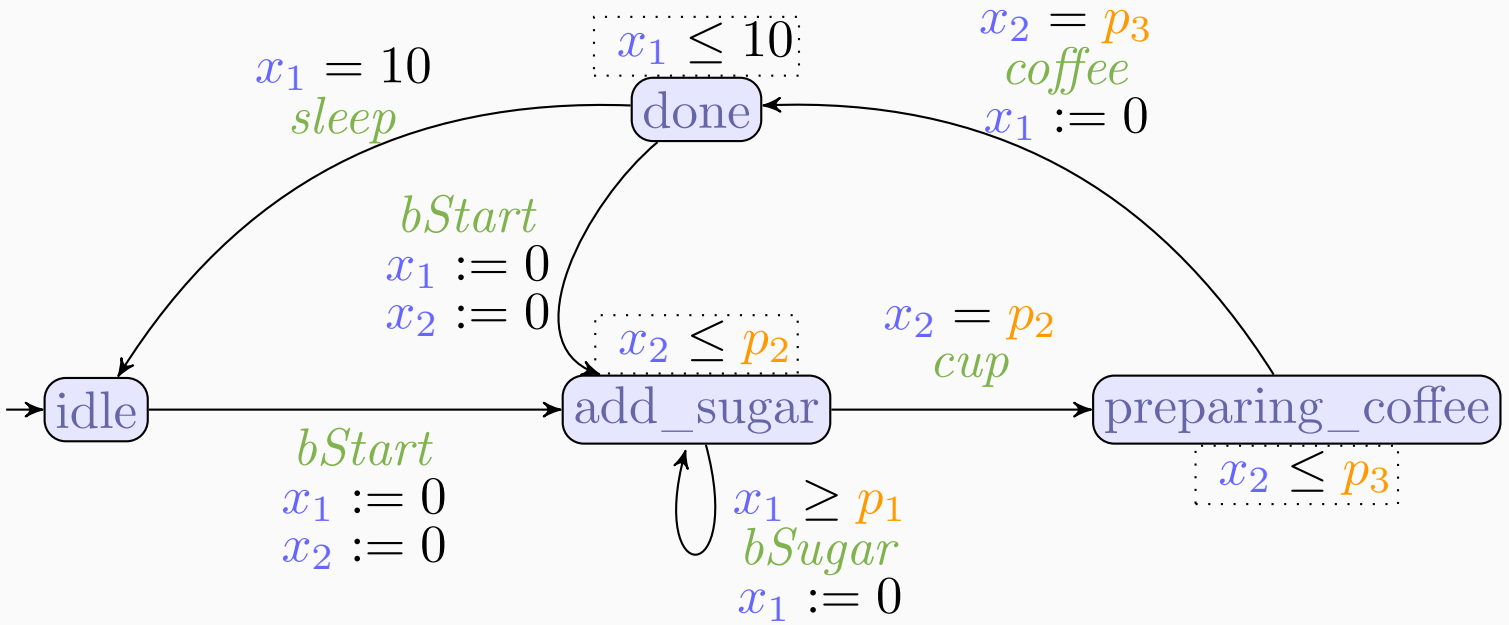
Timed Automata



clocks and constants

Constraints for invariants and guards.

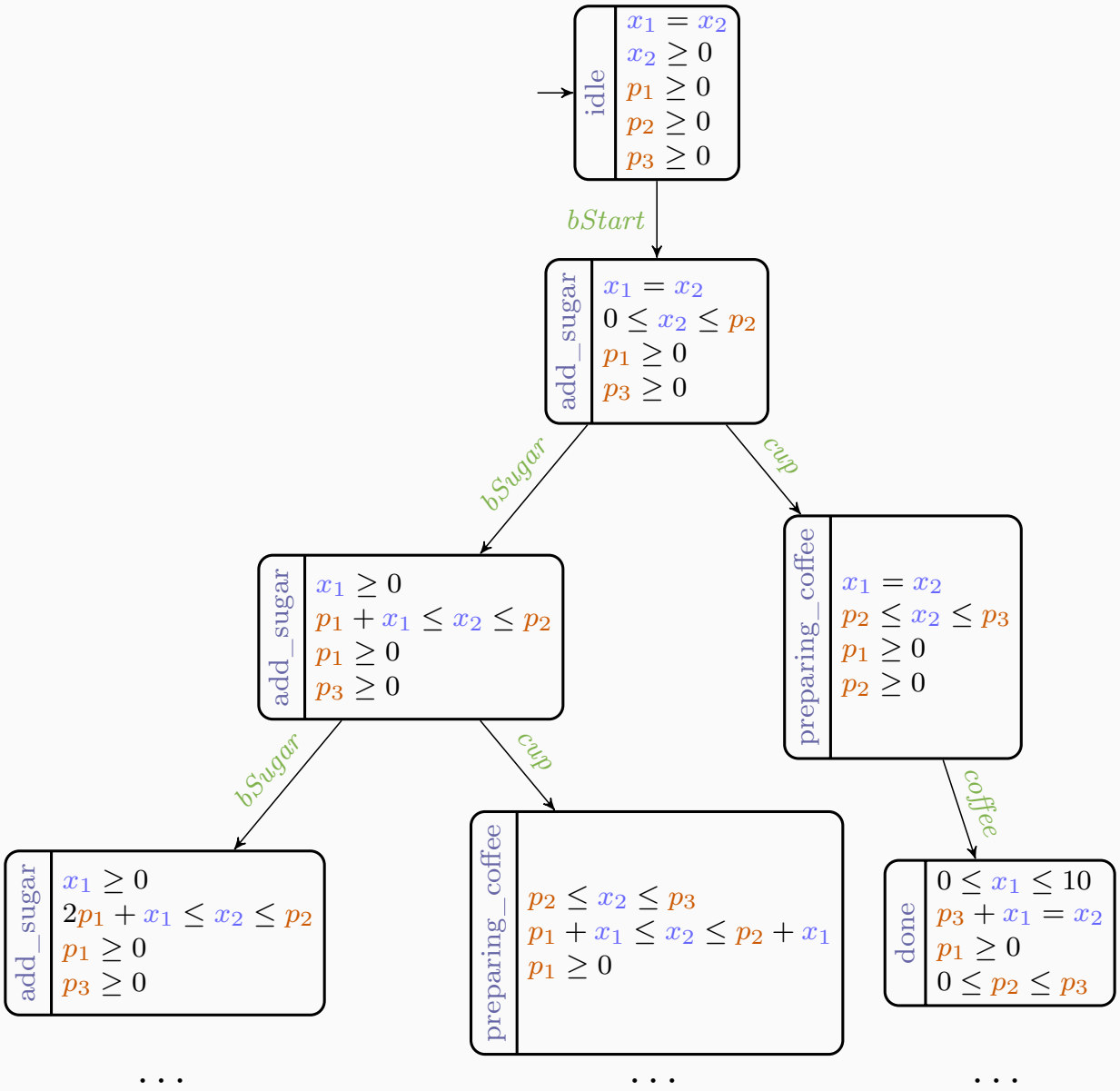
Parametric Timed Automata



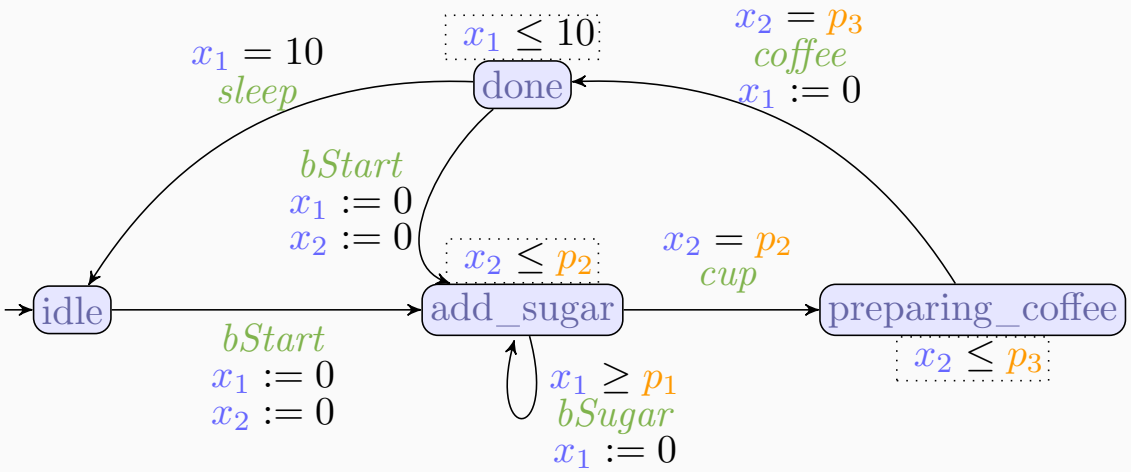
clocks and parameters

Synthesis: If $p_2 \leq p_3$ we can have a coffee!

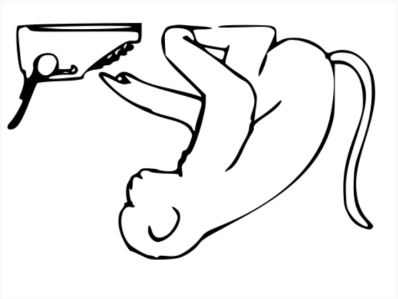
Parametric Zone Graph (PZG)



Parametric Timed Automata



Imitator



- Timed model checking
- Parameter synthesis with dedicated algorithms
- Restricted to PTA-based systems

PTA/Imitator vs Rewriting Logic/Maude

PTA/Imitator

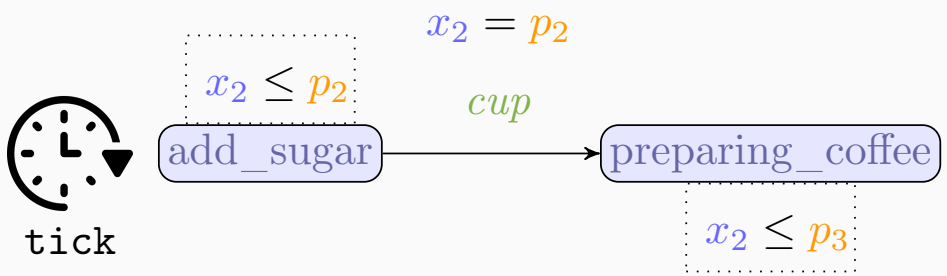
- Heuristic, optimizations and approximation techniques.
- Decidable fragments

RL/Maude

- More expressive (alg. data types)
- Undecidable in general
- No parametric analysis

Learn and take the best from both worlds!

RW semantics for PTA: The tick rule problem

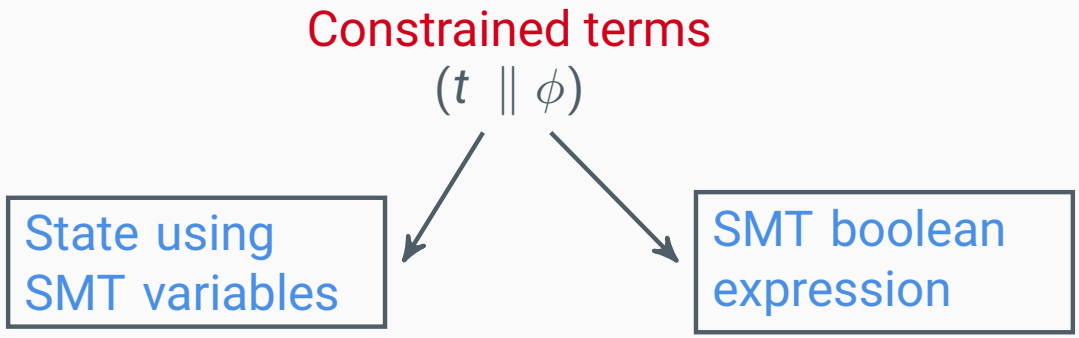



```
vars T P1 P2 P3 : PosRat .
  crl [tick] :
      [ add_sugar : X1 ; X2 ] < P1 ; P2 ; P3 > =>
      < add_sugar : X1 + T ; X2 + T > < P1 ; P2 ; P3 >
      if (X2 + T <= P2 and T >= 0/1) = true [nonexec] .
```

- What is the value of `T`? Not executable rule!
- Parameters P_1, P_2, P_3 are indeed constants (ground rewriting)

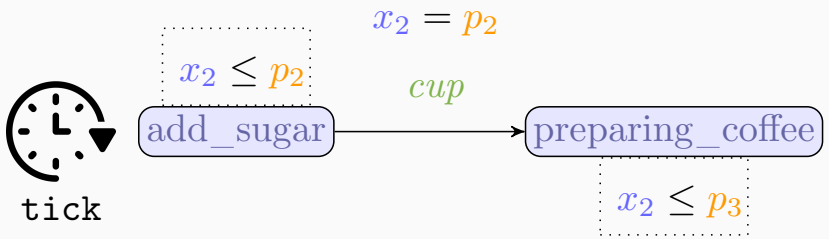
Solution: Symbolic techniques (Rewriting Modulo SMT)

Symbolic semantics: rewriting modulo SMT



- $[\![(t \parallel \phi)]\!]$: possibly infinite set of concrete states (instances of t)
- Symbolic rewrite relation $\rightsquigarrow$: adding constraints + checking satisfiability
- A single symbolic transition captures all possible delays!

RW semantics for PTA



Symbolic states: $t \parallel \phi$ (term + SMT boolean expression)

```
var T P1 P2 P3 : RExpr .  --- SMT Real Variables
crl [add_sugar-tick] :
    [ add_sugar : X1 ; X2 ] < P1 ; P2 ; P3 > =>
    < add_sugar : X1 + T ; X2 + T > < P1 ; P2 ; P3 >
    if smtCheck(X2 + T <= P2 and T >= 0/1) .
```

- Symbolic executions correspond to transitions of the PZG
- Executable in Maude-with-SMT

PTA in Maude: Verification techniques

- Sound and complete reachability analysis:

```
smt-search [1] < idle ; X ; y >   < P1 ; P2 ; P3 >  =>*
               < done ; X' ; X' > < P1 ; P2 ; P3 >
                such that ψ .
```

Parameter synthesis: accumulated constraint $t \parallel \phi$:

- EFϕ synthesis: reachability plus quantifier elimination $\exists X.\phi$
- Only available for the SMT solver Z3.
- AG$\neg\phi$: finding all solution (termination problem!) and negating the result.

Termination Problem

- `smt-search` stop exploring when it sees the same symbolic state.
- A new fresh variable is created when the tick rule is applied.
- No termination for negative queries.

new `reachability` command

Subsumption:

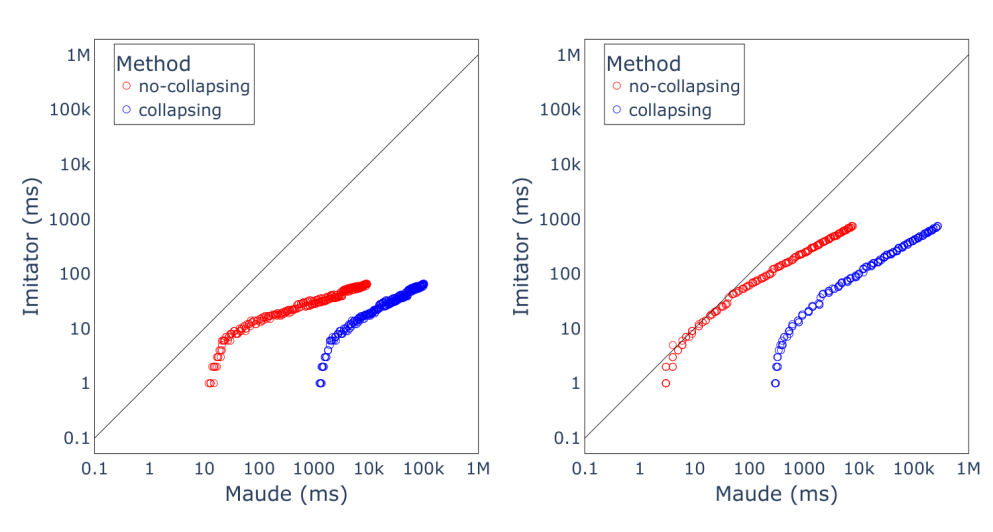
$$\phi_u \parallel t_u \sqsubseteq \phi_v \parallel t_v$$

iff there exists θ s.t. $t_u = t_v\theta$ and the implication $\phi_u \Rightarrow \phi_v\theta$ holds

Theorem

The command `reachability` terminates iff the PZG is finite.

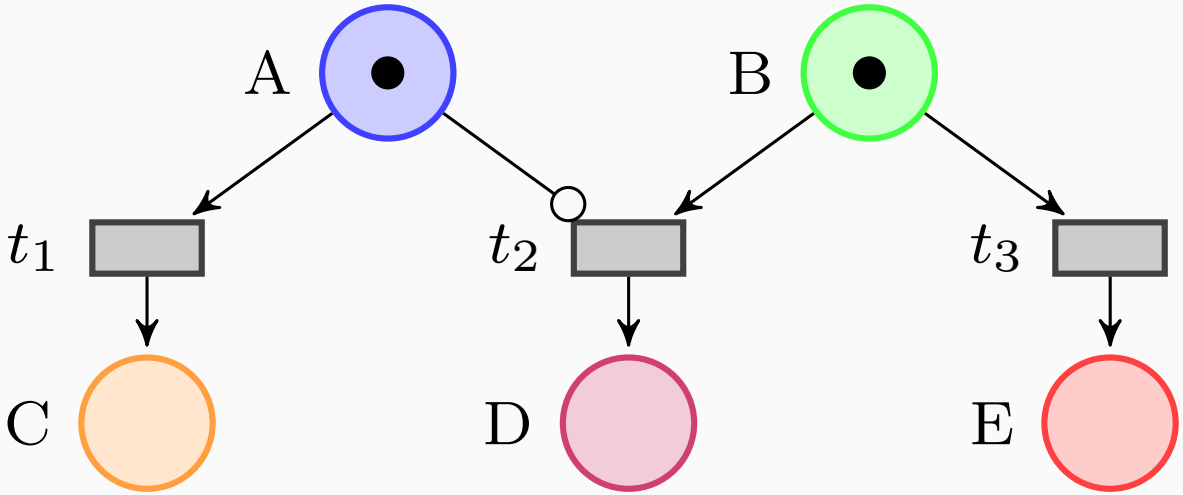
What we have so far...



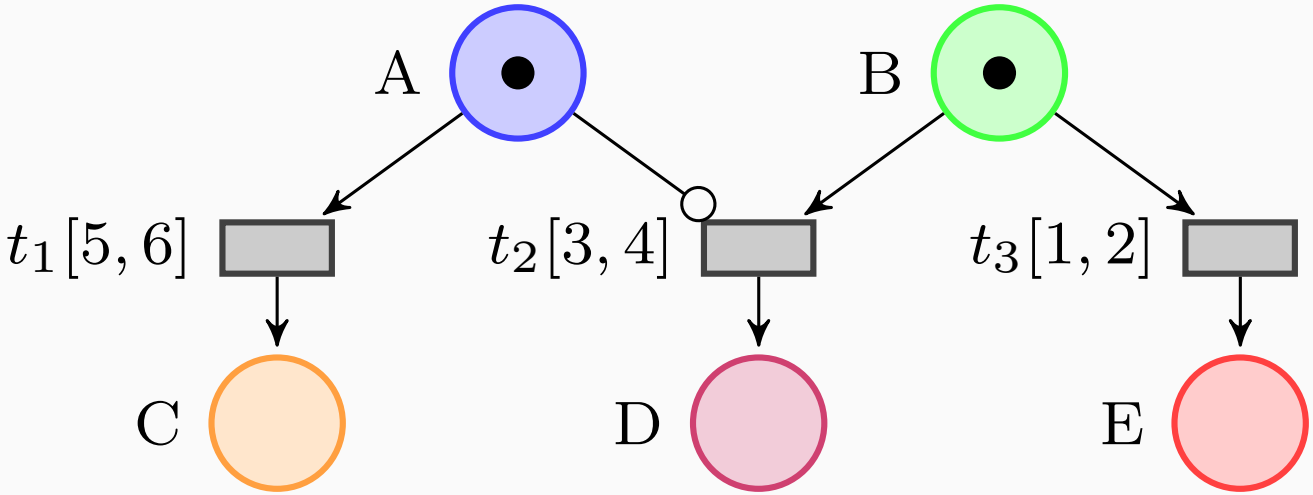
- Relatively simple model (“simple” states)
- Each PTA is compiled into a theory $\mathcal{R}$
- No equations in the theory (a requirement for smt-search)
- Sound and complete reachability analysis
- “Standard” folding was enough for guaranteeing termination
- Parameter synthesis only available with Z3.
- But this is not enough for networks of PTAs with global variables.....

Plan

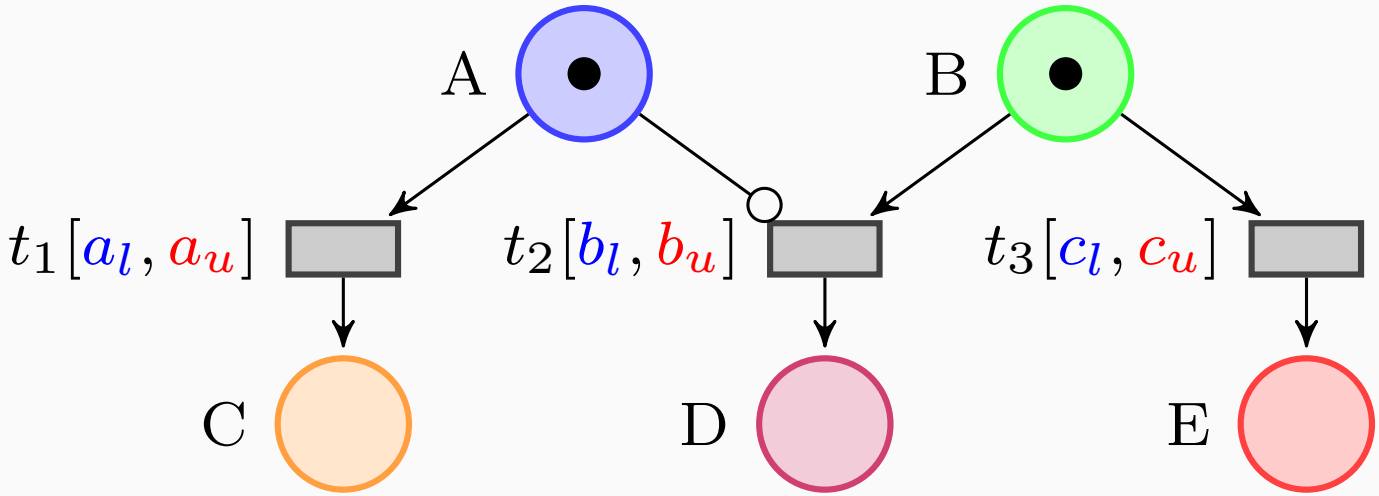
Petri net with inhibitor arcs



Time Petri net with inhinitor arcs



PITPN: Parametric time Petri net with inhibitor arcs



Synthesis problem: finding values for the parameters s.t. certain property holds

Semantics

Concrete semantics: $\xrightarrow{\delta}$; $\xrightarrow{t}$

- $\xrightarrow{\delta}$: time advances $\delta \in \mathcal{R}_+$ and intervals are updated
- δ is constrained so that no enabled transition is missed
- $\xrightarrow{t}$: updates the marking and newly enabled transitions are reset

Symbolic semantics: $\overset{\delta}{\Rightarrow}$; $\overset{t}{\Rightarrow}$

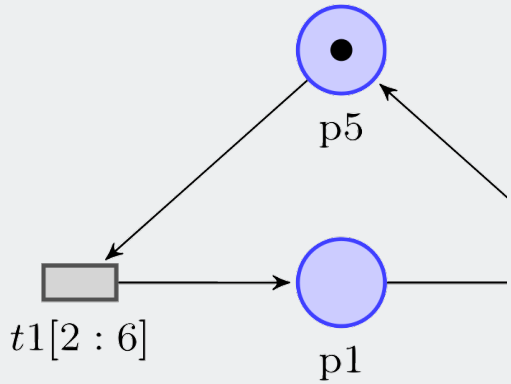
- State classes (M, D)
- D is a constraint (conjunction of inequalities) on parameters Λ

Representing PITPNs in Maude

Syntax and representation: $[\![\mathcal{N}]\!]$

```
--- Sorts and constructors
sorts Label Place Marking ...
op _|->_ : Place Nat -> Marking [ctor] .

--- Example of a concrete PITPN (no parameters)
  "t1" : "p5" |-> 1 -->
         "p1" |-> 1
         inhibit empty in [2 : 6]
```



Dynamics: transition $\xrightarrow{t}$

```
 crl [applyTransition] :
    M  :  (L -> T) ; CLOCKS  :
    (L : PRE --> POST inhibit INHIBIT in INTERVAL) ; NET
=>  (M - PRE) + POST :
    L -> 0 ; updateClocks(CLOCKS, M - PRE, NET) :
    (L : PRE --> POST inhibit INHIBIT in INTERVAL) ; NET'
if active(M, L : PRE --> POST inhibit INHIBIT in INTERVAL)
   and (T in INTERVAL) .
```

Representing PITPNs in Maude

The tick rule: delay transition $\xrightarrow{\delta}$

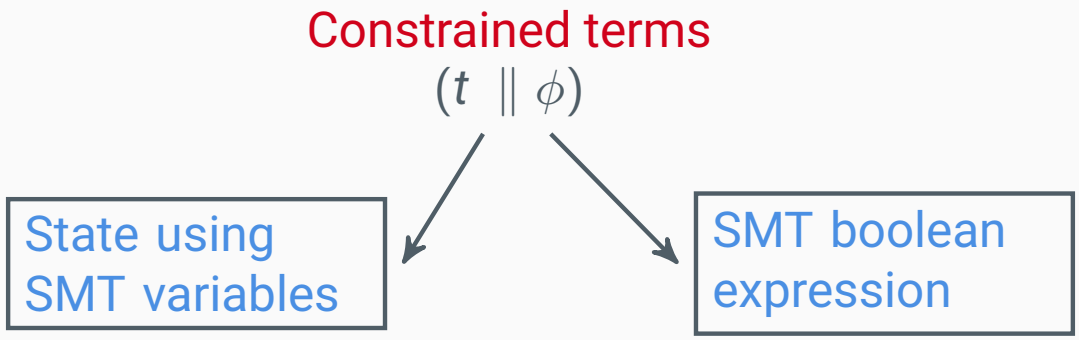
```
crl [tick] : M : CLOCKS : NET  =>  M : increaseClocks(M, CLOCKS, NET, T) : NET
if  T <= mte(M, CLOCKS, NET) [nonexec] .
```

- Enabled transitions are not missed (predicate *mte*)
- Non executable : *T* needs to be sampled

Theorem (Bisimulation)

For any PITPN $\mathcal{N}$, the transition system induced by the rewrite relation in theory $[\![\mathcal{N}]\!]$ is bisimilar to the concrete semantics of $\mathcal{N}$.

Symbolic semantics: rewriting modulo SMT



- $[\![(t \parallel \phi)]\!]$: possibly infinite set of concrete states (instances of t)
- Symbolic rewrite relation $\rightsquigarrow$: adding constraints + checking satisfiability
- A single symbolic transition captures all possible delays!

```
crl [tick] :  tickOk    : M : CLOCKS     : NET => ....
           =>  tickNotOk : M : increaseClocks(M, CLOCKS, NET, T) : NET
if (T >= 0 and mte(M, CLOCKS, NET, T)) .
```

Theorem (Adequacy)

For any PIPTPN $\mathcal{N}$, the symbolic semantics of $\mathcal{N}$ corresponds to $\rightsquigarrow$-transitions.

Termination problem

- Each application of rule `tick` creates a fresh SMT variable
- Maude+SMT analyses do not terminate (even if the PZG is finite)
- Standard subsumption relation is not sufficient

New folding procedure

- New relation $\preceq$: based on matching + existential quantifier elimination
- Let U, V be two symbolic states and $U \Downarrow_{now} = t_u \parallel \phi_u$, $V \Downarrow_{now} = t_v \parallel \phi_v$

$$U \preceq V \text{ iff } t_u = t_v\theta \text{ and } \exists(U \Downarrow_{now}) \Rightarrow \exists(V \Downarrow_{now})\theta$$

- $\exists(U \Downarrow_{now})$ hides the information about ticks.
- Soundness and completeness: $[\![U]\!] \subseteq [\![V]\!]$ iff $U \preceq V$
- Collapsing states: $M \mapsto C_1 \vee \cdots \vee C_n$

Theorem (Termination)

If the symbolic transition system for $\mathcal{N}$ is finite, then so is the resulting symbolic rewrite relation with $\preceq$-folding.

Analyses

EF-synthesis (**EF** ϕ). Standard Maude's search

```
search [1] init(net, m0, φ) =>* S : PHI' || ( TICK : M : CLOCKS : NET )
       such that smtCheck(PHI' and not k-safe(1,M)) .
```

Safety synthesis (**AG**($\neg\phi$)). Search + Folding

```
safety-syn(net, m0, a:Real >= 30/1 and a:Real <= 70/1, k-safe(1,M)) .
```

Strategies. Rewriting + Strategy

New analysis: What happens if t_3 has a higher priority?

```
t3-first := ( applyTransition[L <- "t3"] or-else all )!
```

Analyses

Model Checking. (Search + Maude's LTL model checker)

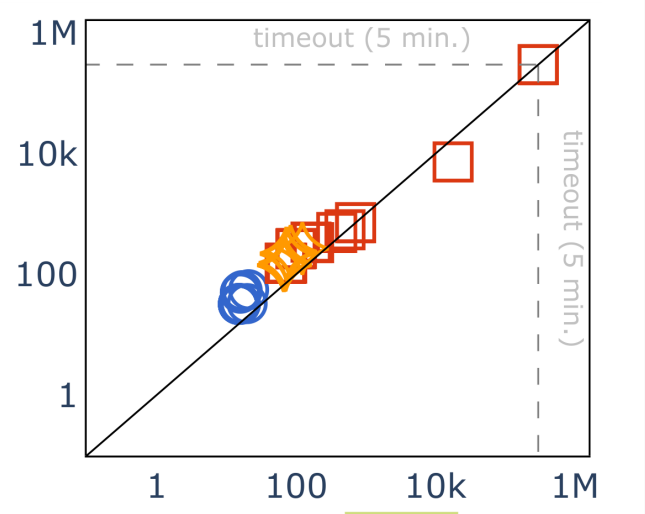
- The TCTL fragment $\exists \mathsf{F}_J\, \phi \mid \forall \mathsf{G}_J\, \phi \mid \phi \rightsquigarrow_{\leq b} \psi$ can be checked with search + folding
- The TCTL fragment $\mathbf{Q}\, \phi\, \mathsf{U}_J\, \psi \mid \forall \mathsf{F}_J\, \phi \mid \exists \mathsf{G}_J\, \phi$ can be checked using Maude's LTL model checker (+ some theory transformations)
- New analysis: full LTL model checking is available to PITPNs

Parametric initial marking

- The number of tokens at a place *p* is an SMT integer variable
- New analysis: Initial marking synthesis

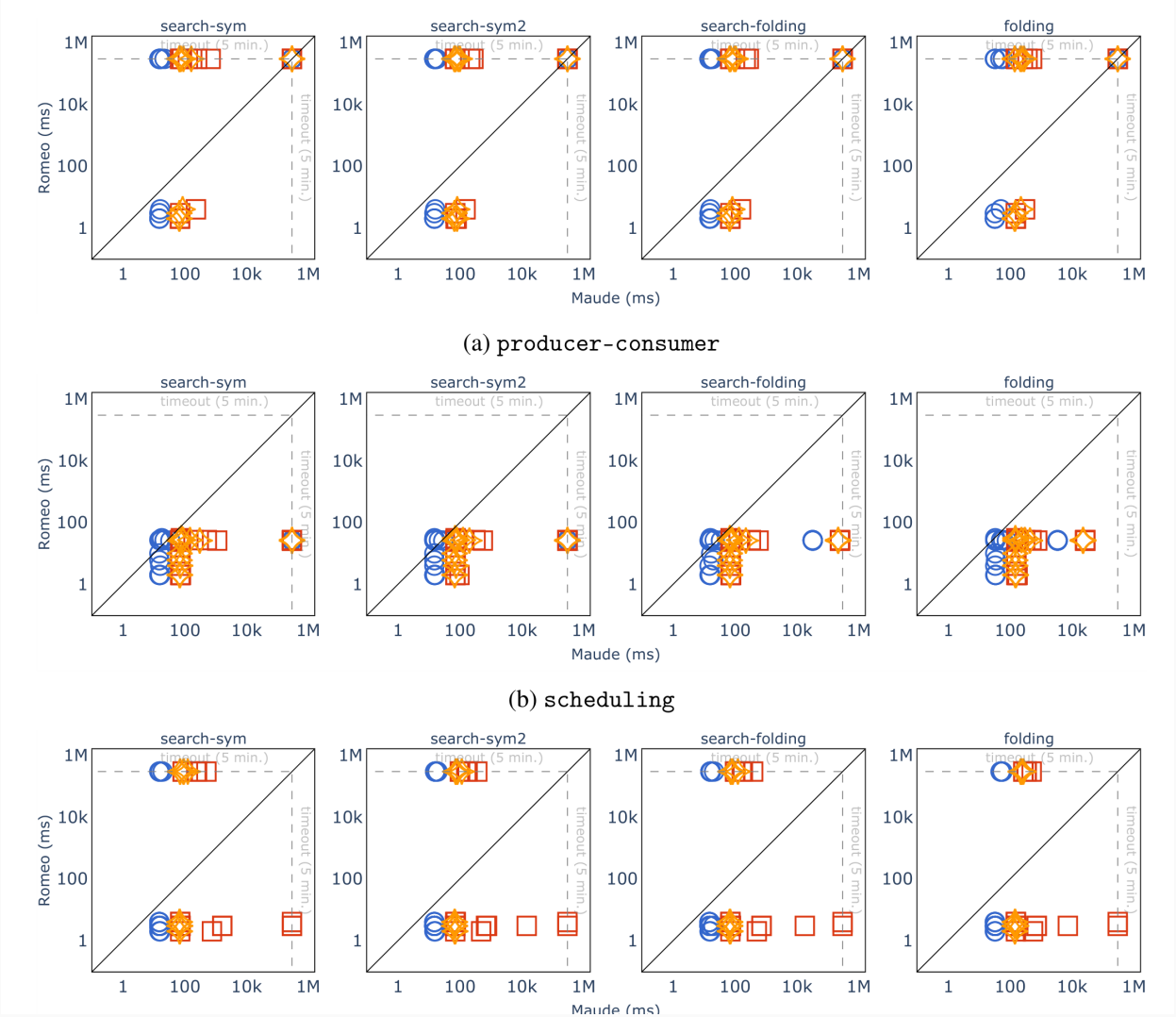
Folding and synthesis

- Quantifier elimination is needed for folding and synthesis.
- Z3 was the only option available for $\exists X.\phi$... but...



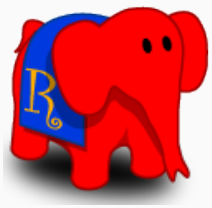
- We have implemented the FME procedure in Maude
- Yices2 (the fastest in our benchmarks) can be used now!

Benchmarks



(a) `producer-consumer`

(b) `scheduling`

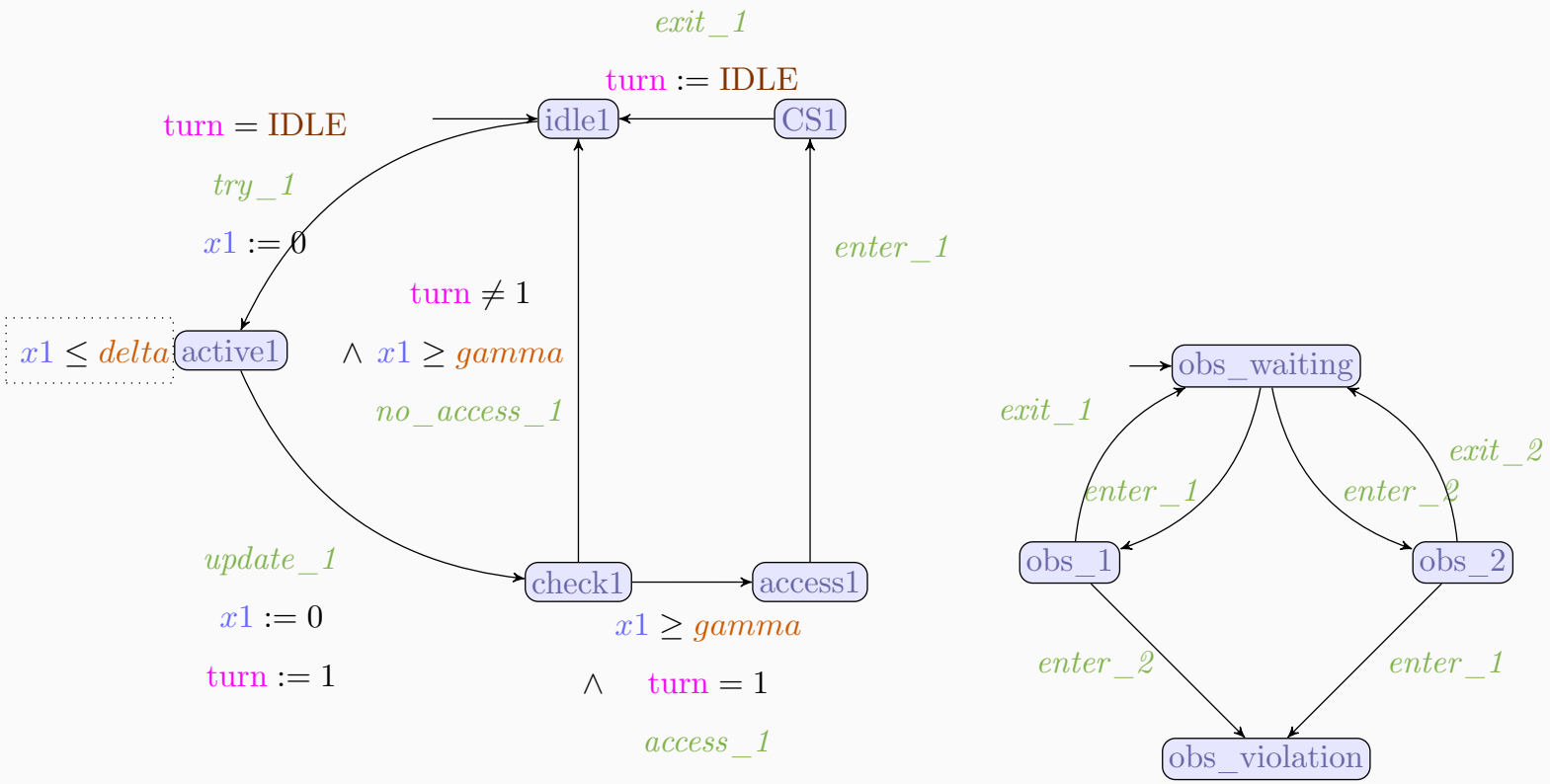


What we have so far...

- Equations are part of the theory and `smt-search` cannot be used directly.
- Standard subsumption does not work.
- A new folding eliminating tick variables is needed.
- Our benchmarks look much better with our own FME procedure (Yices2).

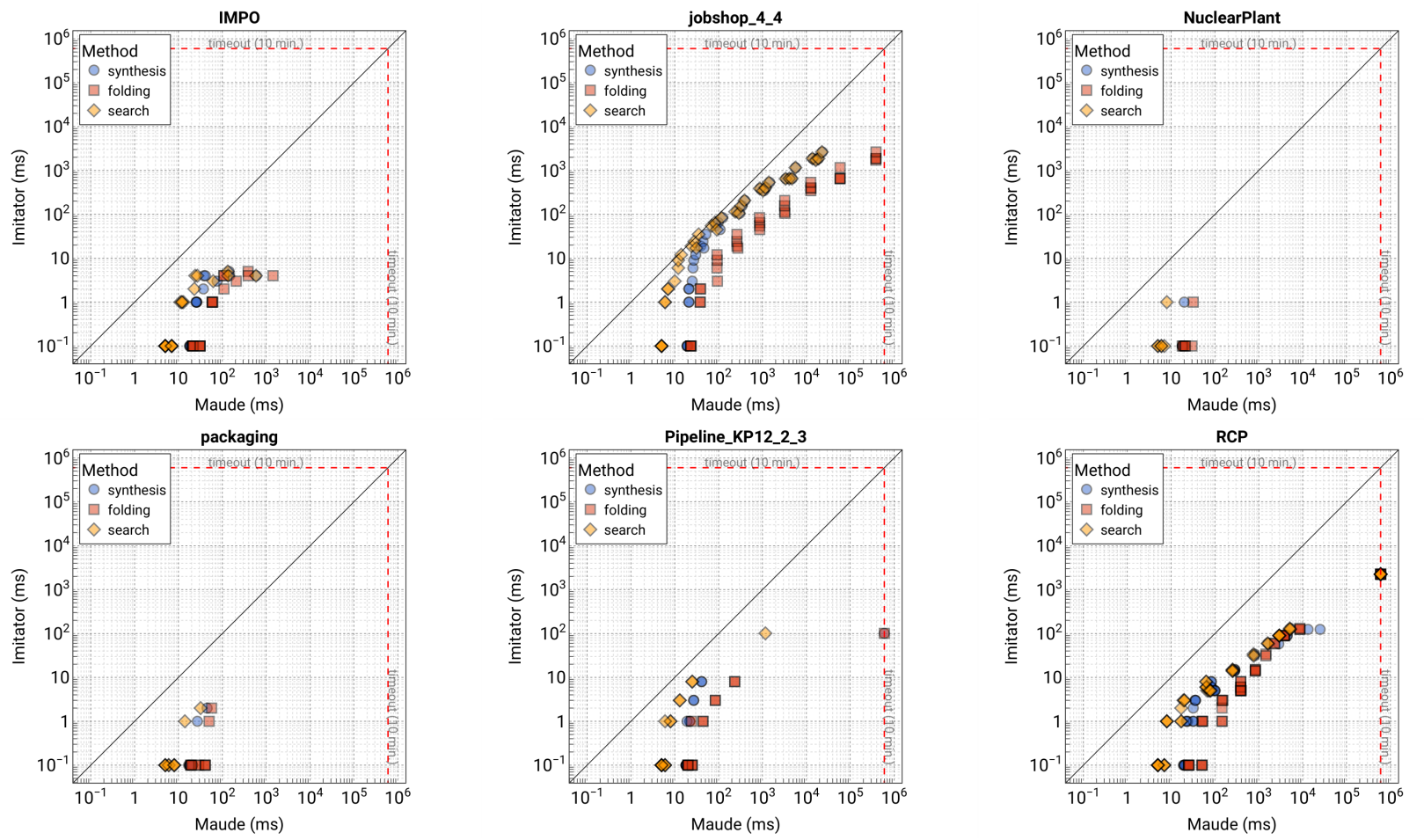
Plan

Networks of PTAs with Variables



- We need an interpreter (product of automata)
- However, the interpreter is not as fast as the “compiled” PTA.

Imitator vs Maude



Plan

Concluding Remarks

- Executable symbolic rewrite semantics for PTAs and PITPNs
- Sound and complete analyses: synthesis, reachability, TCTL model check
- New Analyses:
  - Full LTL model checking
  - Strategies
  - Synthesizing initial markings
- System for quick prototyping of new analysis methods
- Concrete steps for symbolic analysis of real-time rewrite theories
- New strategy language for real-time theories (WRLA'24).
- Full LTL/CTL (symbolic) model checking (in progress).

Thanks!

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